Z notatników Stefana Białoboka

From Stefan Białobok’s notes
ROBERT TOMUSIAK¹, PAWEŁ ZARZYŃSKI²

The old trees trunk’s volume determination with use of theodolite

Określanie miąższości pni sędziwych drzew za pomocą teodolitu

¹Subdivision of Dendrometry and Forest Productivity, Warsaw University of Life Sciences, 159 Nowoursynowska Street, 02-776 Warsaw, Poland
²Department of Forest Protection and Ecology, Warsaw University of Life Sciences, 159 Nowoursynowska Street, 02-776 Warsaw, Poland

Received: 2 August 2007, Accepted: 10 September 2007

ABSTRACT: Tremendous ancient trees have always impressed humans very much. They have been a topic of many stories and legends and also objects of the cult. The fascination with huge trees can be expressed by the willingness to explore their dimensions and to state which tree is the biggest one. That gives the opportunity to make various comparisons. The most often determined parameters include stem girth or diameter (usually measured at the height of 1.30 m), crown diameter and stem volume. Usually, it is not too difficult to measure the previous features, for the measurements can be executed using some simple instruments. However, correct determination of the stem volume is a bigger challenge. In this paper we deal with that problem.

Traditional methods of volume determination can not be accurate for monumental ancient trees of untypical shape. Therefore, we need to search for non-standard, indirect mensuration methods, for example the use of the theodolite. Application of some trigonometric formulae to the results of the traditional measurements of the diameter not that high above the ground (for example at the breast height) and the distance between the stem and the theodolite, enable very precise to determine diameters at any height as well as the height of the tree. The way of doing it is described in the paper: placing the instrument, leveling, centering and measurement of both vertical and horizontal angles. The trigonometric formulae for determination stem characteristics are given. The use of the method is presented on the example of monumental Oak tree for determination of a log volume by Smalian sectional formula.

The described method of indirect tree measurements seems to have a wide spectrum of applications in mensuration of various species as well as various stem shapes and dimensions. The results obtained from theodolite measurements can be used for stem volume determination, not only by application in simple formulae, but also in more precise sectional formulae. The method is rather time-consuming but its use for measurements of monuments of nature gives reason to do it.

Key words: monuments of nature, volume, measurement, theodolite, dendrometry
Introduction

Ancient, tremendous trees always impressed human very much, they were a topic of many stories and legends and also the objects of a cult. A German traveler and naturalist living at the turn of 18th and 19th century Alexander von Humboldt, called them „monuments of nature”, appreciating their uniqueness (Szczęsny 1971). This term is now in an universal use and determines one of the best known forms of nature conservation (Dmochowska 2006).

One of the expressions of fascination with huge trees was willingness to explore their dimensions aiming to state which tree is the biggest one and to have possibility to make various comparisons. On the other hand knowledge of measurements of monumental trees is indispensable from the point of view of their inventory. On the base of various sources (Zarzyński 2003, Dmochowska 2006) we know, that there are between 25 and 33 thousand monuments of nature in Poland, which group together about 105 thousand monument trees, growing as individual trees as well as in groups and alleys. Information about these exceptional creatures of nature included in local and regional registers is very often small and does not contain too many measurements. Additionally, the methods of measurements and its results are often rather questionable.

The most often determined tree parameters are: stem girth or diameter (measured usually at 1,30 m tree height), crown diameter and volume. Measurements of the first features do not cause usually too many difficulties and are possible to execute with the help of some simple instruments, but correct determination of stem volume is a bigger challenge. In the paper we deal with this problem.

Volume of the standing tree is determined most often with the formula:

\[ V = G \cdot H \cdot F \]  

where G is cross-sectional area (usually at breast height), H – total tree height, and F – form factor (usually breast height form factor). The last element of the formula – F – is the coefficient reducing volume of the comparative cylinder (arithmetic product of given cross-sectional area and the tree height) and volume of the examined tree or its part. After transformation of the formula (1) you can see that a form factor is a quotient of volume of a dendrometric block (stem, log) and volume of the corresponding comparative cylinder. Relatively easy to determine are two first volume elements: cross-sectional area and height. The cross-sectional area is obtained from the measurements of diameter or girth at breast height and next used in the formula for circle cross-sectional area. The height of the tree can be determined by hypsometers – instruments very often used in forestry. The third volume’s element – form factor – can be calculated when the volume is known. So a paradox is appearing – for volume determination a coefficient is necessary, which has got the determined volume in numerator. How to deal with it? There is a possibility of solving that problem in forestry taking
data from other trees using three methods: sample trees methods, volume or form factor tables methods and empirical formulae based methods. For the first method we need to cut sample trees. So it could rather be used only in scientific research due to a very huge laboriousness; an advantage is high precision and accuracy. Usually it is used for determining mean value of form factor for the stand. There is no use of this method for determination of form factor for the single tree, in contrary to next two methods. Both tables and empirical equations methods are based on correlations between form factors (or volumes) and some measurement features, in most often cases: height and diameter at breast height. We only need to know two values for reading from tables or calculating by empirical formulae volume of the given tree. For single trees these methods could give sometimes even high errors (Bruchwald 1998) but for the stands the errors are reducing.

So, are we right using these methods to determine the volume of monumental trees? Taking into account that a monument of nature is an inimitable dendrometric solid, with many branches (some of them could be broken or cut), also with many callus, root influxes etc., the answer for that question is negative. A reason for that could be also a fact, that the volume tables and empirical formulae were elaborated on the basis of correlation relationships between various tree characteristics for trees growing in forest. So the volumes or form factors given in the tables and the empirical equations are average values for trees at given measurements.

The shape of trees accredited as monuments of nature significantly diverges from the shape of forest trees. Monuments of nature are very often trees growing in parks or alleys, which means trees growing in totally different light conditions, comparing with forest trees. An additional factor changing stem shape or the whole tree habit is time; the morphological features of ancient trees differ very much from younger trees growing in economic stands, for which the tables and empirical formulae are built.

It can be easily concluded from the information mentioned above, that none of the described groups of volume determination can be accepted unconditionally for ancient trees due to low accuracy and difficult to foresee errors. Therefore, we need to search for non-standard methods, using indirect ways of tree measurements. You can use Bitterlich’s relascope or telerelascope, but more precise measurements will be obtained with the geodesic instruments like for example theodolite. That solution for needs of inventory of trees in park was proposed and described very widely by Olenderek (1984). It was also used for determination of dendrometric features of the monument of nature – an oak named “Mieszko I”, to get a comparative material for estimation of supposed features of a very famous, historic, Lithuanian Oak “Baublis” (Szczepkowski et al. 2002).

The description of the method

As mentioned above, for the volume determination the measurement of basic dendrometric features is required: height and, depending on the formula used,
one or more diameters at various height from the ground level. Obtaining these diameters is not a problem for felled trees, lying on the ground. But absolutely different situation is when a tree is standing and you can not use the caliper on the whole length, especially for ancient trees. A good solution seems to be a theodolite, and an indirect measurement based on its use. Although a theodolite is a rather expensive instrument, it is accessible because many forest, geodesic and construction institutions and companies own it. On the base of traditional measurement of diameter not far from above the ground (for example at breast height) and distance between the stem and the theodolite you can determine very precise diameters at any height, after application of some trigonometric rules.

How to measure a tree with the use of the theodolite? Before the measurement the theodolite must be placed precisely in such a distance from the tree, which allows to observe the stem through the telescope. The distance should not be too small and too big because it can decrease the accuracy of measurement. The optimal distance is the distance equal to about 1,5-2 heights of the tree. Locating the theodolite you must find a place which allows you to set a vertical axis running across the middle of the stem. The next step is leveling and centering horizontal and vertical axes – the both vertical and horizontal scales must be set to 0 (degrees or grades depending on the theodolite’s scale). You must also mark the place of crosscut of the telescopic sight line with the tree. You can do it for example by sticking into the tree’s bark a colorful tack (in that case a help of another person is useful). One notice – because of practical considerations, in special case when a tree is growing on slope or in a waved terrain, the theodolite should be placed in the way, in which the crosscut of the telescopic sight line on the tree will be at the height lower than 1,50 m.

The next step is a measurement of the girth on the marked level in a very careful way, with the tape. It is needed for calculation of the radius, which is the distance from the hypothetical middle of the stem to its periphery. The last measurement done in a traditional way is measurement of the distance between the point marked on the tree’s bark and the base of the theodolite (d). After adding the radius of the stem for that value (d) we receive the distance between the middle of the stem and the base of the theodolite (D), which is needed for indirect measurements of heights and stem diameters.
For determination of the tree’s height (H) the measurement of two vertical angles is necessary (Fig. 1): $\alpha_d$ between marked point on the stem and the base of the stem (for determination of the distance between marked point height and the base of the stem) and $\alpha_g$ between the marked point on the stem and the treetop (for determination of the distance between them). Therefore, after these measurements, we can obtain the tree height after the calculations using the trigonometric equation:

$$H = D \left( \tan \alpha_d + \tan \alpha_g \right) \quad (2)$$

One notice: making the measurements of the vertical angles for determination of the tree height you should set the line of sight not for the middle of the stem, but for its periphery, avoiding errors being a result of a three-dimensional stem structure. The stem periphery on the given height is the place where its hypothetical middle is located.

Using this relationship it is also possible to determine length of any vertical part of stem, above or below of the marked point (Fig. 2), by the formula:

$$H_{1-2} = D \left( \tan \alpha_2 - \tan \alpha_1 \right) \quad (3)$$

This procedure could be very useful in determining stem volume by sectional formulae, described in next part of the paper.

For indirect determination of the stem diameters at any height the measurement of two angles is required – this time horizontal ones (Fig. 3). For this purpose the height of the diameter location should be found by the method given above, leading the telescope’s axis through the stem periphery. After that, the aiming line of the telescope should be directed into the middle of the stem in the place, where
the outgoing plane was prepositioned. Then it is possible to measure angles $\beta_1$ and $\beta_2$, enclosed between that plane and lines running through the stem periphery. Calculation of the diameter at the given height ($R_n$) is possible on the base of angles $\beta_1$ and $\beta_2$, distance between the middle of the stem and the theodolite ($D$) and vertical angle $\alpha_n$ using the formula:

$$R_n = D \cdot \cos \alpha_n \left( \tan \beta_1 + \tan \beta_2 \right)$$

(4)

Diameters at any height and also height of the stem, obtained on the base of indirect measurements, enable to calculate stem volume using any dendrometric formula.

**An example of the method’s use**

The described method of an indirect measurement of some stem dendrometric features can be applied in volume determination of majority monumental trees, with the use of any formula. However simple formulae in most cases could not be accurate because of the complicated and heterogenous objects. Therefore, a better solution seems to be the use of the sectional formula. The idea of that formula is to divide a stem into many sections, and then the use of the simple formula (eg. Smalian or Huber formula) for each section. The stem volume is a sum of volumes of each section.

Mainly three reasons appeal for the choice of a sectional formula for volume determination; firstly – the sections vary in fullness, which is why this simple formula used for each section can give both positive and negative errors. This makes the reduction of the errors possible. The second reason is a higher proportion of upper and lower diameter for each section, comparing to the whole stem. As the proportion is higher the errors are lower, which was proved by analysis of the theoretical accuracy of simple formulae. And the last reason: by dividing stem into many sections we make many measurements, so the possible errors can be also positive or negative, which can result in error reduction (Bruchwald 1998).
A practical example of the use of the theodolite method for volume determination by a sectional formula is the determination of a log volume of an oak, a monument of nature, growing in Radziwiłłów Forest District in a division 19a. We used the Smalian sectional formula for that purpose, dividing the log on seven unequal-length sections to point out irregular profile of the log (Fig. 4). It required the indirect measurements of diameters at lower and upper end of sections as well as section lengths. It allowed us to determine volume of each section, for example for the first section:

\[ V_1 = \frac{1}{2} (G_l + G_u) \cdot L \]  

where:
- \( V_1 \) – volume of the first section
- \( G_l \) – cross-sectional area at the lower end of a section
- \( G_u \) – cross-sectional area at the upper end of a section
- \( L \) – length of a section

After determination of volume for each section and summing up the obtained values we got the total volume of the log as follows:

\[ V = V_1 + V_{II} + V_{III} + V_{IV} + \ldots + V_n \]  

where \( V_1 \) to \( V_n \) are the mean volumes of the sections I to n. The results are presented in table 1. The total volume of the log obtained by this method is 9.458 m³.

**Table 1. – Tabela 1.**  
Log volume determination of the monumental English Oak tree growing in Radziwiłłów Forest District

<table>
<thead>
<tr>
<th>Section number</th>
<th>Lower diameter Średnica dolna (m)</th>
<th>Lower cross-sectional area Przekrój dolny (m²)</th>
<th>Upper diameter Średnica góra (m)</th>
<th>Upper cross-sectional area Przekrój górny (m²)</th>
<th>Section length Długość sekcji (m)</th>
<th>Section volume Miążdżości sekcji (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,29</td>
<td>1,31</td>
<td>1,09</td>
<td>0,93</td>
<td>1,25</td>
<td>1,4000</td>
</tr>
<tr>
<td>2</td>
<td>1,09</td>
<td>0,93</td>
<td>1,19</td>
<td>1,11</td>
<td>1,73</td>
<td>1,7646</td>
</tr>
<tr>
<td>3</td>
<td>1,19</td>
<td>1,11</td>
<td>1,25</td>
<td>1,23</td>
<td>1,27</td>
<td>1,4859</td>
</tr>
<tr>
<td>4</td>
<td>1,25</td>
<td>1,23</td>
<td>1,18</td>
<td>1,09</td>
<td>1,57</td>
<td>1,8212</td>
</tr>
<tr>
<td>5</td>
<td>1,18</td>
<td>1,09</td>
<td>1,15</td>
<td>1,04</td>
<td>0,89</td>
<td>0,9478</td>
</tr>
<tr>
<td>6</td>
<td>1,15</td>
<td>1,04</td>
<td>1,14</td>
<td>1,02</td>
<td>1,39</td>
<td>0,7410</td>
</tr>
<tr>
<td>7</td>
<td>1,14</td>
<td>1,02</td>
<td>0,99</td>
<td>0,77</td>
<td>1,45</td>
<td>1,2978</td>
</tr>
<tr>
<td><strong>Total (Razem)</strong></td>
<td><strong>9,55</strong></td>
<td><strong>9,4583</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The method is rather time consuming, especially if one would like to determine also volume of big branches. But in application for unique trees – monuments of nature – using this method seems to be rational.

Conclusions

1. The described method with the use of theodolite allows to determine any diameter at any stem height, so it seems to have a wide spectrum of applications in measurements of various species as well as various stem shape and dimensions.

2. Accurately obtained stem measurements with the theodolite could be used in stem volume determination by simple formulae addressed to cut trees but also for more precise volume determination – with the use of sectional formulae.

3. As a time consuming method its application could be restricted to measurements of unique, impressive, monumental trees as well as trees growing in parks, alleys, cities. The method could be especially useful for objects with the shape diverging from typical forest trees, for which the traditional methods are elaborated.

References